XX. *A formula for expressing the decrement of human life. In a letter addressed to Sir* EDWARD HYDE EAST, *Bart. M. P. F. R. S. By* THOMAS YOUNG, *M. D. For. Sec. R. S. Communicated February* 2, 1826.

Read April 19, 1826.

MY DEAR SIR,

THE investigation of the laws, by which the general mortality of the human species appears to be governed, is of equal importance to the statesman, the physician, the natural philosopher, and the mathematician; and as you have had occasion to pay particular attention to the subject, I trust that it will not be disagreeable to you to receive the results of an inquiry, into which I have entered, for the purpose of appreciating, if not of reconciling, the many discordant opinions that have been advanced, respecting the comparative mortality of mankind, at different times, and under different circumstances.

Of late years, there is little doubt, that, whether from the protective effects of vaccination in infancy, or from the increase of the comforts of the poorer, and of the temperance of the more affluent classes of society, or in some measure also from the simplification of the practice of physic and surgery, there is a decided increase in the mean duration of life in many parts of Europe: but it is also extremely probable that this improvement has been greatly exaggerated; partly on account of the limited description of the persons on whom

the observations have been made, and partly from an erroneous opinion respecting the profits of certain establishments, which have been attributed to the employment of too low an estimate of mortality, while they have, in fact, been principally derived from the high rate of interest which the state of public credit has afforded.

A very laborious and well informed actuary has lately asserted, before a Committee of the House of Commons, that "the duration of existence now, compared with what it was a hundred years ago, is as four to three, in round numbers." (Parl. Rep. N. 522, p. 44.) It does indeed happen, that this particular result may in one sense be very correctly deduced from the immediate comparison of the annual mortality of a certain number of persons of the same description, that is, annuitants, at the periods in question; nor is it possible to deny that some importance must be attached to the remark: but the mortality of the same class of persons in France, at the earlier period, was no greater, according to Mr. DEPARCIEUX'S estimate of their longevity, than in England at the later, while the general mortality in France has never been materially less than in England, and appears at present to be even somewhat greater: and it can only be conjectured, that the annuitants of the tontine of King WILLIAM were in general most injudiciously selected, while those who were the subjects of Mr. DEPARCIEUX'S observations, like the annuitants of the modern tontines, were chosen with more care, or with greater success. Mr. FINLAISON'S tables, therefore, though they may be extremely just and valuable for the purpose of setting a price upon annuities to be granted on the lives of the proposers, cannot, with any prudence, be adopted where

the parties concerned have an interest in offering the worst lives that they can find; notwithstanding any partial security that might be afforded by the exercise of medical skill in their rejection; and if it is true, that some of the tontines were principally filled by lot (Rep. p. 16), with the children of country clergymen and magistrates, it must still be supposed that the families of such persons may have been more healthy than the average of the population of London and the country taken together.

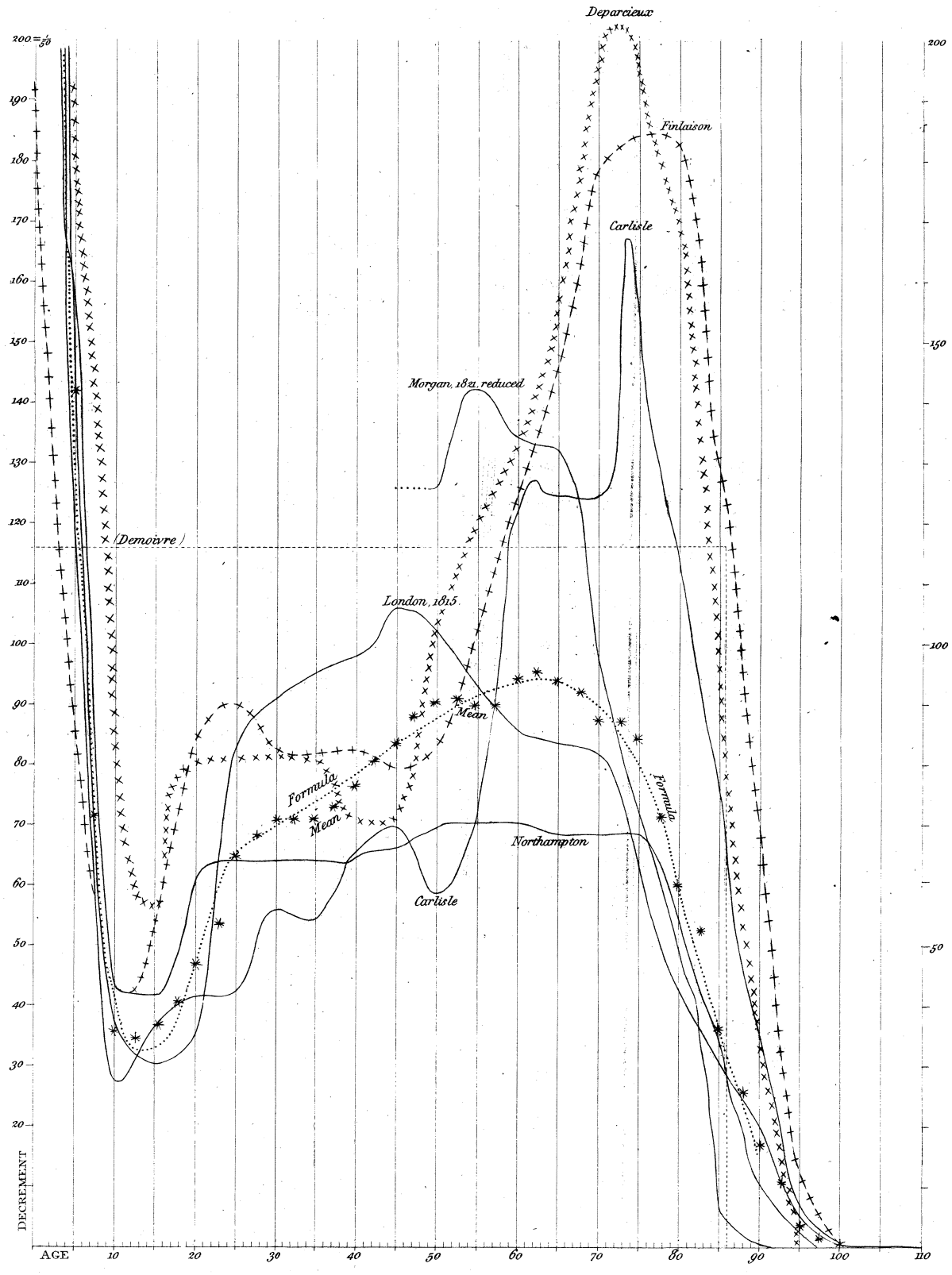
For the comparison of the general characters of different tables of mortality, the simplest and most obvious criterion is perhaps the number of individuals out of which one dies annually, which is also the number of years expressing the expectation of life at the time of birth. But this test is liable to material objections with regard to the most usual application of the table, which depends more on the comparative expectations at later periods than in early infancy. For example; the Northampton table affords results, throughout the whole of middle and advanced life, agreeing almost exactly with DEMOIVRE's hypothesis of equal decrements, although the *annual mortality* is supposed to be nearly 1 in 25 at Northampton, instead of 1 in 43, as assumed by DEMOIVRE. It would therefore be very unjust for a person allowing the truth of DEMOIVRE's hypothesis, to condemn the practical employment of Dr. PRICE's tables in common cases, on account of this variation only. A less exceptionable test will be, to find the mean of the numbers expressing, for different ages, the *full term* of life, or the sum of the age and twice the expectation, taking the decads from 10 to 80 as the most important. Another standard of comparison may be the age

which is equal to the expectation of life, and which, in DEMOIVRE's arithmetical hypothesis, is the *mean age* of all the population, and probably very near it in all tables formed from actual observation. In this manner a general comparison of the most remarkable tables may be instituted.

Characteristics of Mortality.

	Annual mortality one in	Mean full term of life.	Mean age.
Roman estimate of Ulpian, probably with some deduction for present value.....	.....	74 (+ disc^t)	26 (+ disc^t)
Deparcieux's Tontines, beginning 1689	47.67	94.17	32.5
Halley's table for Breslau, 1690.....	33.50	87.15	28.1
Tontine of 1695, Finlaison, males.....	37.61	83.42	27.25
——————————— females	(43.0)	87.50	29.32
Simpson's table for London, about 1730	19.2	82.30	25.7
Dupré, in Buffon, about 1750	33.0	85.30	28.67
Northampton tables, about 1760.....	25.18	87.39	28.86
Swedish tables, about 1785.....	36.12	91.86	31.3
France, before the Revolution, Duvillard	28.76	86.96	29.0
Finlaison's tontine and annuitants, about 1800, males.....	50.16	93.25	32.0
females	55.51	100.7	34.6
Finlaison's Chelsea Pensioners	.....	90.0	29.65
Carlisle tables, about 1810, Milne.....	37.14	95.47	32.6
Returns for all England, 1811.....	49		

The order of the mortalities expressed by the first column of this table, is, SIMPSON, Northampton, France, DUPRE', HALLEY, Sweden, Carlisle, tontine 1695 males, females, DEPARCIEUX, returns of 1811, tontines of 1800, males, and females; the order of the second column is SIMPSON, tontine of 1695, males, DUPRE', France, HALLEY, Northampton, females 1695, pensioners, Sweden, tontines of 1800, males, Carlisle, females of 1800: but besides this difference in the order, the disproportion exhibited in this column is less enormous than in the former; the numbers of the Carlisle


Deparcieux
Finlaison
Carlisle
Morgan, 1821, reduced
(Demoivre)
London, 1815
Mean
Formula
Mean
Formula
Northampton
Carlisle
200 = 1/50
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
DECREMENT
200
150
100
50
AGE
10
20
30
40
50
60
70
80
90
100
110

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tables, for example, exceeding those of the Northampton by one half in the former, and by one tenth only in the latter. The proportion of Mr. FINLAISON's tontines also stands as 3 to 4 in the first, and as 7 to 8, or 8 to 9 only in the second: the latter comparison giving a much fairer practical estimate of the comparative longevity, indicated by the tables, than the former.

Another mode of easily appreciating the regularity and the analogies of different tables is, to construct a diagram, in the form of a curve, of which the absciss represents the age, and the ordinates the corresponding decrements of life. (Plate XI.) The inspection of such a diagram is sufficient to convince us of the great irregularity of the Carlisle tables of mortality, which must obviously have been formed, as they confessedly were, from observations on a very limited number of individuals, so that they exhibit a succession of different climacterics, after which the mortality is diminished, while about the age of 74 the curve that represents them towers to an incredible height, affording an expectation of longevity which some of the strongest advocates of those tables have abandoned in their practical applications, since they take their estimate of life, in advanced age, even lower than it is represented in the Northampton tables.

It appears therefore to be highly probable, that the fairest basis for general computations, to be applied throughout Great Britain, may be obtained by a proper combination of the tables of Northampton, which have been long known and very generally approved, with the Carlisle tables, corrected however in their extravagant values of old lives, by some other documents; and with the mortality of London as

derived from the parish registers, which, when thus incorporated with tables formed in the country, will be freed from the objections that have been made to the observations of burials in great cities only.

The Carlisle table agrees in the earlier parts pretty nearly with the observations of Mr. MORGAN on the experience of the Equitable Office from 1768 to 1810, as it appears from Mr. MILNE's comparison, as well as from the reduction and interpolation of those observations published by Mr. GOMPERTZ in the Philosophical Transactions for 1825: but for correcting the later portions of the Carlisle table, it may be allowable to employ a subsequent register of the experience of the Equitable Office, so far as it is possible to make any inferences from it with safety.

The numbers of deaths occurring in 20 years, as recorded by Mr. MORGAN, might have been made the foundation of a very valuable determination of the mortality occurring in a certain class of persons, if the number of the Equitable Society had become stationary before the commencement of the record: but in order to deduce from it a just estimate of the value of life, it would then be necessary to alter the numbers of deaths at each age, in the inverse proportion of the numbers of the living compared: that is to say, not simply of the sums of the persons admitted under that age, but of the numbers of persons born whom they represent: since, in comparing the joint mortalities of any two lists of persons, we must obviously add together the deaths belonging, not to a given number of persons of various ages, but of a number proportionate to the survivors at the respective ages out of a given number of births, so that in this manner the apparent mortality in the

earlier portions of the register would require to be augmented, not only on account of the smaller number of persons who have actually contributed to furnish it, but also on account of the greater proportion that these persons bear to the corresponding number at birth, when compared with the survivors at more advanced ages, who represent a population still more exceeding their own numbers. On the other hand, since the register in question relates only to a limited number of years, immediately following a very rapid increase of the Society, it is evident that the deaths must have occurred at earlier ages than if it had been continued until all the lives had dropped.

Of these three modifications, it may be sufficiently accurate for the present purpose to omit the two latter as nearly counterbalancing each other, and to augment the earlier numbers in the proportion only of the members of the Society to whom they must necessarily have belonged, supposing that the admissions had taken place about the same ages at all periods; assuming also the number of survivors at 45 to be in the same proportion to the births as in the Carlisle table. We may then proceed to take a mean between the mortality thus obtained, with proper interpolations, and the observations at Carlisle, as the second of the three principal bases to be afterwards incorporated with the mortality of Northampton and of London. Further than this, it is impossible to place any great reliance on Mr. MORGAN's document, which makes the annual deaths, in "a population exceeding 150000," not quite 1 in 1500.

Of the mortality of London, taken for the ten years from 1811 to 1820, it may be observed, that its results bear the

internal evidence of greater apparent correctness than either of the other bases, exhibiting a curve less irregular in its flexures, and generally intermediate between the others: it has also the advantage of exhibiting the duration of life as prolonged by the general introduction of vaccination: and when thus incorporated with the registers of two places in the country, each reduced to an equal supposed population, it must probably be sufficiently corrected for the errors that may be attributed to the effect of an afflux of settlers at an early age. The mean obtained in this manner might be employed at once as a standard table without much inconvenience, but it still exhibits some minute but obvious irregularities, as an inspection of the line of stars in the diagram will show, principally perhaps from the want of skill or care with which the interpolations have been made by Dr. PRICE and others. The most effectual of all interpolations for *harmonizing* the various orders of differences, is to obtain a formula which shall extend with sufficient accuracy throughout the whole curve. It may be easily believed that it must be extremely difficult to find such an expression; and that its form must be too complicated to be applied to any practical purpose throughout its extent. I have however drawn a curve which comes extremely near to the line of stars, and crosses it in 10 or 12 different points, by means of the equation $y = 368 + 10x - 11.(156 + 20x - xx)^{\frac{3}{2}} + \frac{1}{285 + 2.05xx + 2\left(\frac{x}{10}\right)^6} - 5.5\left(\frac{x}{50}\right)^{10} + \frac{5.5^2}{4000}\left(\frac{x}{50}\right)^{20} - 5500\left(\frac{x}{100}\right)^{40}$: y being the number of deaths among 100000 persons, in the year that completes the age x.

The terms of this formula have some remarkable relations to the different periods of life. HALLEY's first approximation was $y = 1000$, throughout life. DEMOIVRE's arithmetical hypothesis was $y = \frac{100000}{86} = 1163$: but of the present formula the principal foundation, as extending to the whole of life, is, $y = 368 + 10x$. In infancy the term containing the reciprocal of the powers of x has a preponderating value: in youth, the term $-(156 + 20x - xx)^{\frac{3}{2}}$, which diminishes the mortality, ends somewhat abruptly at 25, and would be incapable of being employed with safety in algebraical calculations, from its having a negative as well as a positive value. Old age is expressed almost exclusively by the high powers at the end of the formula, which terminate the series with great and increasing rapidity. It is obvious that for many purposes of calculation, the terms belonging to youth and to old age might be neglected without inconvenience, and that, for the middle portion of life, the terms $368 + 10x$ alone, with some little modification, might be employed as sufficiently correct; or certainly as much nearer to the truth than either the arithmetical or geometrical hypothesis of DEMOIVRE. The relations of the different parts of the formula will be best appreciated from their developement in the following tables.

DOCUMENTS.

Deaths among persons assured by the Equitable Society, for 20 *years, from* 1800 *to* 1821.—Morgan on Assurances, 1821, p. 325.

Age.	Deaths.	Assured.	Deaths cor.	Interpolations.			
10 to 20	7	1494		251	514	631	630
20 to 30	37	8996		275	566	636	630
30 to 40	166	33850		299	606	646	630
40 to 50	299	45429	630	323	616	655	630
50 to 60	458	36489	649	347	618	663	630
60 to 70	536	19042	604	371	620	665	
70 to 80	345	6454	359	395	622	663	
above 80	82		82	419	624	657	
				440	626	644	
		151754		470	628	630	(820)
				3590	6040	6490	20090
						Carlisle	45070

Mortality of London from 1811 *to* 1820.—Gent. Mag.

	Between 0 & 2	2 & 5	5 & 10	10 & 20	20 & 30	30 & 40	40 & 50	50 & 60	60 & 70	70 & 80	80 & 90	90 & 100	100	101	102	103	104	105	108	109	111	113
1811	5106	1638	654	509	1231	1641	1741	1591	1385	1038	449	56	1	1	1	1						
1812	5636	1907	655	620	1226	1685	1841	1543	1425	1193	492	71	0	0	1							
1813	5167	1733	604	526	1108	1501	1751	1606	1559	1211	489	61	1	1	1	0	0	0	0	2	0	1
1814	5845	2038	770	649	1268	1678	1950	1810	1747	1343	592	88	1	1	1	0	0	0	1	0	1	
1815	5200	1916	870	677	1425	1824	2075	1886	1621	1221	674	167	2	1	0	1						
1816	5400	1960	845	675	1464	1912	2123	1955	1720	1308	781	168	3	0	0	1	1					
1817	5698	2019	929	706	1364	1795	1983	1788	1614	1224	683	156	7	0	0	0	0	2				
1818	5381	1815	808	703	1453	1884	2040	1864	1585	1271	722	175	1	1	1	0	0	0	1			
1819	4779	1771	826	631	1577	1990	2095	1918	1600	1230	666	144	0	0	0	1						
1820	4758	1975	887	667	1484	2006	2069	1878	1632	1208	662	119										
	52970	18772	7848	6363	13600	17916	19668	17839	15888	12247	6210	1205	16	5	5	4	1	2	2	2	1	1
													(39)									190565

Interpolations.

Age.	Deaths.	Age.	Deaths.	Age.	Deaths.	Age.	Deaths.	Age.	Deaths.	Age.	Deaths.
0	32970	20	710	40	1855	60	1622	80	529	100	16
1	20000	21	770	41	1885	61	1615	81	768	101	5
	52970	22	960	42	1925	62	1607	82	718	102	5
2	8500	23	1160	43	1963	63	1600	83	678	103	4
3	6000	24	1460	44	1990	64	1592	84	638	104	2
4	4272	25	1680	45	2010	65	1585	85	597	105	2
	18772	26	1700	46	2020	66	1577	86	556	106	1
5	2800	27	1710	47	2020	67	1570	87	516	107	1
6	1800	28	1720	48	2010	68	1563	88	475	108	1
7	1400	29	1730	49	1990	69	1557	89	435	109	1
8	1008		13600		19668		15888		6210		38
9	840	30	1741	50	1959	70	1550	90	380	110	1
	7848	31	1752	51	1920	71	1478	91	290	111	0
10	740 ?	32	1763	52	1880	72	1405	92	180	112	0
11	660	33	1774	53	1840	73	1333	93	90	113	0
12	615	34	1786	54	1800	74	1261	94	70		1
13	605	35	1798	55	1760	75	1188	95	57		
14	600	36	1809	56	1720	76	1116	96	48		
15	603	37	1820	57	1680	77	1044	97	40		
16	610	38	1831	58	1650	78	972	98	30		
17	620	39	1842	59	1630	79	900	99	20		
18	640		17916		17839		12247		1205		
19	670										
	6363										

Comparative Decrements from various Tables.

Age.	North-ampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Eq. Office.	London Bills.	General Mean.	Living.
0	25751	15390			17301	19481	99124
1	11734	6820			10493	9682	79643
2	4309	5050	—	—	4460	4606	69961
3	2876	2760			3148	2928	65355
4	1691	2010			2242	1981	62327
5	1579	1210			1469	1419	60346
6	1202	820			945	989	58927
7	944	580	—	—	725	750	57938
8	687	430			529	549	57188
9	515	330			441	429	56549
10	446	290			389	375	56120
11	429	310			346	362	55745
12	429	320	—	—	323	357	55383
13	429	330			318	359	55026
14	429	350			315	365	54667

Comparative Decrements from various Tables.

Age.	North-ampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Equi. Office.	London Bills.	General Mean.	Living.
15	429	390			317	379	54302
16	455	420			320	398	53923
17	497	430	—	—	325	417	53525
18	541	430			335	435	53108
19	575	430			352	452	52673
20	618	430			372	473	52221
21	644	420			404	489	51748
22	644	420	—	—	503	522	51259
23	644	420			608	557	50737
24	644	420			766	610	50180
25	644	430			882	652	49570
26	644	430			892	655	48918
27	644	450	—	—	897	664	48263
28	644	500			902	682	47599
29	644	560			907	704	46917
30	644	570			913	709	46213
31	644	570			919	711	45504
32	644	560	—	—	925	710	44793
33	644	550			931	708	44083
34	644	550			937	710	43375
35	644	550			943	712	42665
36	644	560			950	718	41953
37	644	570	—	—	955	723	41235
38	644	580			961	728	40512
39	644	610			967	740	39784
40	652	660			974	762	39044
41	661	690			990	780	38282
42	669	710	—	—	1010	796	37502
43	669	710			1030	803	36706
44	669	710			1044	808	35903
45	669	700	1346	(765)	1055	830	35095
46	669	690	1346	(821)	1059	850	34265
47	669	670	1346	(873)	1059	867	33415
48	669	630	1346	(916)	1055	880	32548
49	678	610	1346	978	1044	900	31668
50	695	590	1346	968	1028	897	30768
51	704	620	1375	997	1007	903	29871
52	704	650	1404	1027	987	906	28968
53	704	680	1416	1048	966	906	28062
54	704	700	1421	1060	945	903	27156
55	704	730	1416	1073	924	900	26253
56	704	760	1399	.....	902	894	25353
57	704	820	1381	.....	881	895	24459
58	704	930	1359	.....	866	904	23564
59	704	1060	1348	.....	856	921	22660

Comparative Decrements from various Tables.

Age.	North-ampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Equit. Office.	London Bills.	General Mean.	Living.
60	704	1220	1342	.....	851	945	21739
61	704	1260	1338	.....	848	950	20794
62	695	1270	1333	.....	844	948	19844
63	695	1250	1329	.....	840	941	18996
64	687	1250	1325	.....	836	936	18055
65	687	1240	1321	.....	832	936	17119
66	687	1230	1317	.....	828	929	16183
67	687	1230	1295	.....	824	925	15254
68	687	1230	1210	.....	819	909	14329
69	687	1240	1098	.....	817	889	13420
70	687	1240	1005	.....	813	874	12531
71	687	1340	940	.....	776	867	11657
72	687	1460	895	.....	738	868	10790
73	687	1560	844	.....	700	863	9922
74	687	1660	792	.....	661	858	9059
75	687	1600	742	.....	623	839	8201
76	661	1560	690	.....	585	790	7362
77	627	1460	639	.....	548	741	6572
78	584	1320	587	.....	510	683	5831
79	548	1280	536	.....	472	643	5148
80	541	1160	480	.....	435	599	4505
81	515	1120	440	.....	402	549	3906
82	489	1020	300	.....	376	508	3357
83	472	940	200	.....	356	466	2849
84	412	840	100	.....	335	406	2383
85	351	780	60	.....	313	361	1977
86	291	710	40	.....	292	319	1616
87	241	640	30	.....	271	284	1297
88	180	510	20	.....	251	234	1013
89	137	390	10	.....	230	190	779
90	103	370	8	.....	199	164	589
91	86	300	7	.....	152	130	425
92	69	210	7	.....	94	87	295
93	60	140	6	.....	47	60	208
94	43	100	6	.....	37	44	148
95	26	70	5	.....	30	31	104
96	9	50	5	.....	25	19	73
97	0	40	5	.....	21	14	54
98	..	30	4	.....	16	9	40
99	..	20	4	.....	10	6	31

Comparative Decrements from various Tables.

Age.	North-ampton.	Carlisle.	Equitable Office Red.	Mean of Carlisle and Eq. Office.	London Bills.	General Mean.	Living.
100	..	20	4	.....	8	6	25
101	..	20	3	.....	3	5	19
102	..	20	3	.....	3	5	14
103	..	20	2	.....	2	4	9
104	..	10	2	.....	[1]	2	5
105	..	0	1	.....	1	1	3
106	..	..	0	.....	.5	(.25	(2.00
107	..	..	....	.....	.5	.25	1.75
108	..	..	....	.....	.5	.25	1.50
109	..	..	....	.....	.5	.25	1.25
110	..	..	....	.....	.5	.25	1.00
111	..	..	....	.....	...	.25	.75
112	..	..	....	.....	...	.25)	.50
113	..	..	....	.....	...	.25)	.25)
114	..	..	....	.....	...	.0	.00)

Decrements of Mortality computed from the Formula.

Age $(x-1)$	$368 + 10x$	$-.11\,(156+20x-xx)^{\frac{3}{2}}$	$+\frac{1}{2.85+2.05\,xx+2\left(\frac{x}{10}\right)^6}$	Decrement.
0	378	−255	+ 20408	20531
1	388	241	9009	9106
2	398	313	4695	4780
3	408	359	2805	2854
4	418	386	1848	1880
5	428	409	1322	1341
6	438	427	968	979
7	448	440	746	752
8	458	447	592	603
9	468	451	477	494
10	478	447	392	423
11	488	440	329	377
12	498	427	278	349
13	508	409	238	337
14	518	386	205	337

Decrements of Mortality computed from the Formula.

Age $(x-1)$	$368 + 10x$	$-.11(156+20x-xx)^{\frac{3}{2}}$	$+\frac{1}{2.85+2.05xx+2\left(\frac{x}{10}\right)^6}$	$-5.5\left(\frac{x}{50}\right)^{10}$	Decrement.
15	528	359	178	.....	347
16	538	313	156	.....	381
17	548	291	136	.....	393
18	558	255	119	.....	422
19	568	214	104	.....	458
20	578	174	93	.....	497
21	588	130	82	.....	540
22	598	89	72	.....	581
23	608	51	64	.....	621
24	618	19	57	.....	656
25	628	0	50	.....	678
26	638	.....	44	.....	682
27	648	.....	39	.....	687
28	658	.....	34	.....	692
29	668	.....	30	.....	698
30	678	.....	27	.....	705
31	688	.....	24	.....	712
32	698	.....	21	.....	719
33	708	.....	18	.....	726
34	718	.....	16	.....	734
35	728	.....	14	.....	742
36	738	.....	13	.....	751
37	748	.....	11	—0	759
38	758	.....	10	.4	768
39	768	.....	9	.6	776
40	778	.....	8	.7	785
41	788	.....	8	.9	795
42	798	.....	7	1.2	804
43	808	.....	6	1.5	813
44	818	.....	5	1.9	821
45	828	.....	5	2.3	831
46	838	.....	4	3.0	839
47	848	.....	4	3.9	848
48	858	.....	3	4.5	857
49	868	.....	3	5.5	866
50	878	.....	3	6.7	874
51	888	.....	2	8	882
52	898	.....	2	10	890
53	908	.....	2	12	898
54	918	.....	2	14	906

Decrements of Mortality computed from the Formula.

Age $(x-1)$	$368 + 10x$	$*+\frac{1}{2.85+2.05xx+2\left(\frac{x}{10}\right)^6}$	$-5.5\left(\frac{x}{50}\right)^{10}$	$+.001\left(\frac{5.5\left(\frac{x}{50}\right)^{10}}{2}\right)^2$	$-5500\left(\frac{x}{100}\right)^{40} =$	Decrement.
55	928	2	17	....	....	913
56	938	1	20	....	....	917
57	948	1	24	....	....	923
58	958	1	28	....	....	929
59	968	1	33	....	....	934
60	978	1	39	+ 0	....	938
61	988	1	46	1	....	942
62	998	1	55	1	....	943
63	1008	1	64	1	....	944
64	1018	1	75	1	....	943
65	1028	.....	—88	2	....	942
66	1038	.....	102	3	....	939
67	1048	.....	119	4	....	933
68	1058	.....	137	5	....	926
69	1068	.....	159	6	....	915
70	1078	.....	183	8	....	903
71	1088	.....	211	11	....	888
72	1098	.....	242	15	....	871
73	1108	.....	277	19	....	850
74	1118	.....	317	25	....	826
75	1128	.....	359	32	....	801
76	1138	.....	412	42	....	768
77	1148	.....	470	55	....	733
78	1158	.....	532	71	—0	697
79	1168	.....	604	91	1	654
80	1178	.....	684	117	1	610
81	1188	.....	772	145	2	559
82	1198	.....	872	190	3	513
83	1208	.....	984	242	6	460
84	1218	.....	1108	307	9	408
85	1228	.....	1243	386	14	357
86	1238	.....	1399	490	22	307
87	1248	.....	1567	614	37	258
88	1258	.....	1756	771	52	215
89	1268	.....	1963	963	86	178
90	1278	.....	2192	1201	139	148
91	1288	.....	2444	1493	212	125
92	1298	.....	2713	1849	333	101
93	1308	.....	3032	2300	496	80
94	1318	.....	3371	2840	734	53
95	1328	.....	3744	3504	1041	27
(96	1338	.....	4150	4306	1746	0
97	1348					
98	1358					
99	1368					
100)	1378					

MEAN STANDARD TABLE OF THE DECREMENTS OF LIFE IN GREAT BRITAIN, 1824.

Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.	Age.	Decrement.	Living.
0	20531	100003	30	705	46527	60	938	21810	90	164	589
1	9106	79472	31	712	45822	61	942	20872	91	130	425
2	4780	70366	32	719	45110	62	943	19930	92	87	295
3	2854	65586	33	726	44391	63	944	18987	93	60	208
4	1880	62732	34	734	43665	64	943	18043	94	44	148
5	1341	60852	35	742	42931	65	942	17100	95	31	104
6	979	59511	36	751	42189	66	939	16158	96	19	73
7	752	58532	37	759	41438	67	933	15219	97	14	54
8	603	57780	38	768	40679	68	926	14286	98	9	40
9	494	57177	39	776	39911	69	915	13360	99	6	31
10	423	56683	40	785	39135	70	903	12445	100	6	25
11	377	56260	41	795	38350	71	888	11542	101	5	19
12	349	55883	42	804	37555	72	871	10654	102	5	14
13	337	55534	43	813	36751	73	850	9783	103	4	9
14	337	55197	44	821	35938	74	826	8933	104	2	5
15	347	54860	45	831	35117	75	801	8107	105	1	3
16	381	54513	46	839	34286	76	768	7306	106	.25	2
17	393	54132	47	848	33447	77	733	6538	107	.25	1.75
18	422	53739	48	857	32599	78	697	5805	108	.25	1.50
19	458	53317	49	866	31742	79	654	5108	109	.25	1.25
20	497	52859	50	874	30876	80	610	4454	110	.25	1.0
21	540	52362	51	882	30002	81	559	3844	111	.25	.75
22	581	51822	52	890	29120	82	513	3285	112	.25	.50
23	621	51241	53	898	28230	83	460	2772	113	,25	.25
24	656	50620	54	906	27332	84	408	2312	114	0	0
25	678	49964	55	913	26426	85	357	1904			
26	682	49286	56	917	25513	86	307	1547			
27	687	48604	57	923	24596	87	258	1240			
28	692	47917	58	929	23673	88	215	982			
29	698	47225	59	934	22744	89	178	767			

I shall take this opportunity of endeavouring to demonstrate, in a simple and undeniable manner, the error into which Dr. PRICE and his followers have fallen, in consequence, as it appears, of their adopting the legal restraints on usury as essential steps in the mathematical calculation of the amount of compound interest. The error has indeed of late

years been very commonly admitted; but its effects have not been so satisfactorily rectified as could be desired.

In the 66th volume of the Philosophical Transactions, for the year 1776, we find a Paper of Dr. PRICE, in which he lays down these theorems, r denoting the interest of £1. for a year, and n the term or number of years during which any annuity will be paid, p the perpetuity, or $\frac{1}{r}$, y the value of an annuity paid yearly, and h half yearly: then, I, $y = p - \frac{1}{r(1+r)^n}$; and, II, $h = p - \frac{1}{r\left(1+\frac{r}{2}\right)^{2n}}$: and as examples, taking $r = .04$, and $n = 5$, we have $y = 4.4518$, and $p = 4.4913$.

Now, if we analyse the results thus obtained, by dividing them into the present values of the separate payments, they will stand thus:

I. Present value of £1. payable at the end of 1 year.	£.	.961538
2 years		.924556
3 years		.888996
4 years		.854804
5 years		.821927
		4.451821
II. Present value of 10 shillings, payable at the end of half a year	£.	.49020
1 year		.48058
1½		.47127
2		.46192
2½		.45286
3		.44398
3½		.43528
4		.42674
4½		.41837
5		.41018
		4.49138

The present values of 10 shillings are therefore assumed;

	I,	II,
at 1 year	.48077;	.48058
2 years	.46228	.46192
3 years	.44450	.44398
4 years	.42740	.42674
5 years	.41096	.41018

The latter column exhibiting obviously a larger deduction for discount than the former; so that the rate of interest in the two calculations is by no means the same: although in the case of $r = .05$, they would respectively represent the highest rate of interest allowed by our laws to be received without a new investment or engagement: but this arbitrary restraint ought certainly not to affect the mathematical consideration of the question.

The difficulty, if any person thinks it such, may be avoided by a mode of investigation which I have lately had occasion to point out. "An annuity, of which a payment is due on a given day, is more valuable than an annuity purchased on that day, and to commence a year after, by the amount of a year's payment: and *the value of a life annuity, becoming payable at any intermediate time between the day of purchase and its first anniversary, will be greater than the simple tabular value of the annuity by a sum proportional to the anticipation of the payment*;" the increase of the value being very nearly uniform, when we suppose the anticipation to be gradually increased: this increase of the value comprehending obviously the greater probability as well as the greater proximity of each payment, and proceeding from day to day by very nearly equal increments. Thus, if we wished to purchase an

annuity of £100. a year, and its value were £1000., upon the ordinary supposition of the payments commencing after the end of a year; supposing that we desired to have the first payment made at the end of nine months, and the subsequent payments at annual intervals as usual, we should have to add £25. to the purchase money, making it £1025. at whatever rate of interest the value might have been computed. If we began at six months, £50., and if at three months, £75. must be added to the purchase: it being obvious that an additional £100. would be equivalent to an anticipation of twelve months, or to an immediate payment of a year's annuity.

From this simple and incontestable principle, in which the second differences only are neglected, it is very easy to deduce the values of annuities, payable at intervals shorter than a year. An annuity of 1, payable half yearly, is equal to two annuities of $\frac{1}{2}$, the one beginning as usual at the end of the year, the other anticipated by half a year; and the value of this portion is greater than the other by half of one of the payments, that is, by $\frac{1}{4}$: so that "*We may always find the value of a life annuity payable half yearly, by adding a quarter of a year to the tabular value of the same annuity.*"

In a similar manner it is very easily shown, that "*for quarterly payments, we must add* $\frac{3}{8}$ *of a year's value to the computation made on the supposition of annual payments;*" and "the continual bisection of the interval would at last afford us *the addition of half a yearly payment for the value of a daily or hourly payment of a proportional part of the given annuity.*"

"It may also be observed, that when we reckon at 3 per

cent. interest, an annuity payable half yearly is the same, throughout the middle of life, that would be granted on the life of a person a year older, if payable annually."

If it is required to ascertain the value of a reversionary annuity payable half yearly or quarterly, the calculation becomes in appearance a little paradoxical; for since the true value of a reversionary annuity for the life of one person, for example, after the death of another, is the difference between the values of two annuities on the single life and the joint lives, and since an equal addition must be made to these values in consideration of the period of payment being shortened, it follows that the reversionary annuity must be of equal value in either form. This conclusion would indeed be strictly true if the periodical times of payment remained unaltered, according to the supposition from which the value of the annuities is deduced; while in fact it is usual to grant such an annuity to commence at the first quarterly, half yearly, or annual period after the contingent event: a variation which would have no sensible effect in the case of daily payments, but which lessens the value of reversionary annuities at other periods by that of half a payment for the given period, reduced to the present time in the same manner as any other sum assured as payable upon the same contingency of survivorship.

The simplicity observable in the progression of the values of annuities, calculated according to the values of lives here supposed, and at 3 per cent. interest, leads us to inquire what would be the exact law of mortality required to make that progression strictly uniform throughout life; and it will appear on investigation, that in order to have the value $24.45 - \frac{1}{4}x$,

x being the age of the person, which is nearly true between 20 and 70, the annual mortality must be expressed by $\frac{.03x + .066}{100.8 - x}$: a fraction which at 20 becomes $\frac{1}{121}$, at 40, $\frac{1}{48}$, at 60, $\frac{1}{22}$, and at 80, $\frac{1}{8.4}$. Our table gives respectively $\frac{1}{103}$, $\frac{1}{50}$; $\frac{1}{23}$, and $\frac{1}{7.3}$: the Northampton $\frac{1}{71}$, $\frac{1}{48}$, $\frac{1}{25}$, and $\frac{1}{7.4}$. Mr. FINLAISON'S male annuitants $\frac{1}{87}$, $\frac{1}{73}$, $\frac{1}{32}$, and $\frac{1}{8.3}$. The healthiness of Mr. FINLAISON'S annuitants about 40 and 50 is one of the most remarkable features of his table: he observes (p. 58), that out of 10,000 persons at 23, 141 will die in a year, and 141 will die out of the same number at the age of 48; but at the age of 34 there will only die 124. The curve marked by obelisks, +, in the diagram, will show the comparative progress of mortality in this system; which, however valuable the data may be, appears to exhibit too many novelties, if not anomalies, to be generally adopted with confidence: while the line of crosses, ×, representing the tontine of DEPARCIEUX, will serve to show how little difference the lapse of a century has made in the results of these two similar cases.

I shall conclude, my dear Sir, with a comparison of the climacteric years, as they may be called without impropriety, in which the greatest numbers of adults die, as taken from different tables.

I sincerely hope that these considerations may help to undeceive the too credulous public, who have of late not only received some hints that tend to insinuate the probability of an occasional recurrence of a patriarchal longevity, but who have been required to believe, upon the authority of a most respectable mathematician, that the true and unerring value

of life is not to be obtained by taking an average of various decrements, but by adopting the extreme of all conceivable estimates, founded only on a hasty assertion of Mr. MORGAN, and unsupported by any detailed report; an estimate which makes the grand climacteric of mankind in this country, not a paltry fifty four, or the too much dreaded sixty three; but no less than EIGHTY TWO! An age to which nearly one sixth of the survivors at ten are supposed to attain!

Climacterics, or greatest Decrements.

Berlin, formerly.... 38	Breslau, 1695 61	Montpellier, 1782...67	Deparcieux 73
London, about 1733.40	Formula 63	Duvillard, France...67	Carlisle 74
Paris, formerly.....40	Brandenburg.....65	Sweden, 1762.....68	Ackworth, 1752....75
Stockholm, 1762....42	Warrington, 1777 .. 65	Chester, 1776 68	Kersseboom.....77
London, 1764.....43	Norwich, 1765..... 66	Sweden, 1785 69	Finlaison.....78
London, 1815 46		Holycross, 1760 70	E. O. Mr. B.....82
Northampton, 1757. 56			

Believe me, dear Sir,

Your faithful and obedient Servant,

THOMAS YOUNG.

Park Square, 28 Feb. 1826.